DESIGN AND ESTIMATED WEIGHT OF A CANTILEVER BRIDGE

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1914

THE DESIGN AND ESTIMATED WEIGHT OF A

CANTILEVER BRIDGE

A THESIS

PRESENTED BY

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TO THE

PRESIDENT AND FACULTY

OF

ARMOUR INSTITUTE OF TECHNOLOGY

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN CIVIL ENGINEERING

HAVING COMPLETED THE PRESCRIBED COURSE OF STUDY IN

CIVIL ENGINEERING

- 1914 -

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PREFACE.

To demonstrate the fact, that the principles involved in analyzing a Cantilever Bridge are of a most elementary character, a complete set of values for stresses in the members of such a structure has been made. The "Method of Coefficients", which considers a load of unity concentrated at each panel point, was the plan adopted for the work. Since the analytical and graphical determination for stresses are self-explanatory, little explanation for the theory employed has been offerred.

L.Z.

O.S.

1 June '14

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on
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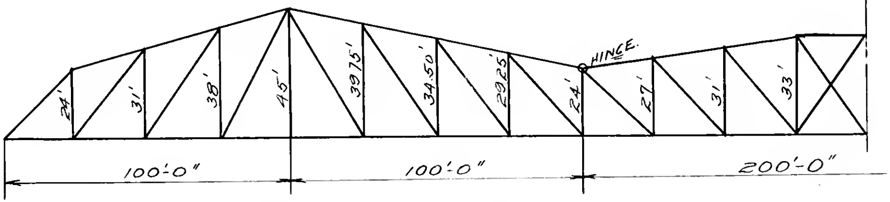
DIAGRAMATIC SKETCH OF CANTILEVER BRIDGE.



(I) Anchor Span = 4 Panels @ 25'-0" = I00'-0"

(2) Cantilever Span = 4 Panels @ 25'-0" = I00'-0"

(3) Suspended Span = 8 Panels @ 25'-0" = 200'-0"

Total length = 2 x (I) + 2 x (2) + (3) = 600'-0"

(Sketch #I)

INTODUCTION.

Cantilever Bridges.

The Idea of the Cantilever Bridge:- The idea was first developed in the attempt to avoid the disadvantages of continuity. In a continuous bridge a slight elevation or depression of one support causes great changes in reactions and stresses. If, however, the chords be cut near the inflection points for full load, the inflection points points for partial loads will occur there also, and thus the reactions will be statically determinate.

Classification:- A cantilever structure may be built as a deck or through bridge. Generally they have three spans; a simple style being the one considered in this thesis. Here the truss is supported at the piers upon a single pin. Then there is the cantilever structure which has two points of support at the pier. In this case there are no diagonals in the panel over the pier, this is to avoid the continuity

that would otherwise exist.

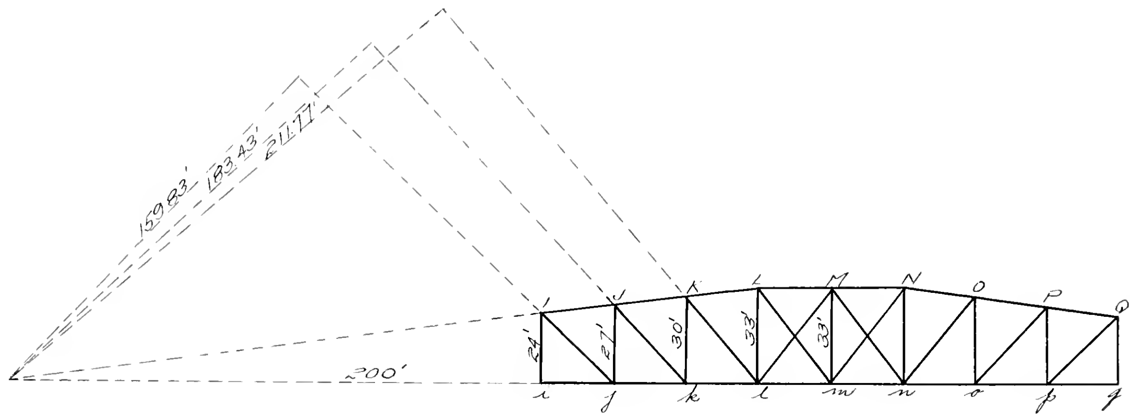
In either of these forms a load between the piers causes a negative reaction at the support, and this the greater the shorter the length of the shore span. To balance the negative reaction which may be caused by the live load, it is necessary that the truss be anchored at the abutment. The end span is hence called an anchor span or anchor arm. Thus for the bridge chosen for this Thesis ae is the anchor span. The cantilever and suspended spans are respectively ei and iz. The suspended truss which connects the ends of the two cantilever arms is merely a simple truss supported at its ends. It is noted then, that it receives no stresses except those due to the loads upon its own floor, such being transmitted to the ends of the cantilever arms exactly as if these ends were abutments.

*Historical Notes:- The idea of building out cantilever beams from opposite shores of a stream and then bridging the interval between them by a

simple beam is an old one. An ancient structure of this kind in Japan is described in Von Nostrand's Magazine for January, 1886, and one in Thibet of 112ft. in length is illustrated in THOMAS POPE'S Treatise on Bridge Architecture, published in New York 1811. This curious book of POPE is largely devoted to a design of his own, called the "flying pendant lever bridge" which was to be built out from the opposite shores until the cantilever arms met at the middle, these arms being anchored by huge abutments. With such a structure he proposed to bridge the Hudson river at New York, the span being about 3000 feet. His scheme met with little encouragement, and it was indeed an impractical one.

* From Merriam and Jacoby on ROOFS & BRIDGES.

DIAGRAMATIC SKETCH OF SUSPENDED SPAN.



(Sketch #2)

SUSPENDED SPAN

Dead Load Stresses:- The suspended span of this cantilever bridge consists of eight panels at 25'-0",totaling 200'-0". As indicated in the diagram of the preceding page there is a rise of 3'-0" in the upper chord of each panel, barring the middle ones which have parallel chords.

Consider a panel load of one kip concentrated at each panel point. Reaction =4Kips. Then let y equal the distance to the intersection of the upper and lower chord from point i. y= 24x25 ÷ 3 = 200'-0"

Tan. α =I.2 α = 6°-5I'
Cos. α = .99286
Sec. α =I.007I9

ij=0

Aa=-3.5

-Chord Stresses.-

IJ=-(3.5x25) ÷ (27x.9929) = -3.26

JK=-(3.5x50)+25 ÷ (30x.9929)=-5.04

jk= 3.5x25 ÷ 30 = 3.24

KL=-(3.5x75)+50+25 ÷ (33x.9929)=-5.73

kl= 3.5x50 - 25 ÷ 30 = 5.0

LM=-(3.5xI00) + (75+50+25) ÷ 33 = -6.06

lm= (3.5x75) - (50+25)

Diagonals.

Tan.Iji = 24÷25 = .96 Iji = 43°-50'
Sin.Iji = 0.69256

Tan.Jkj = 27÷25 =I.08 Jkj = 47°-I2'
Sin.Jkj = 0.73373

Tan.Klk = 30÷25 =I.20 Klk = 50°-I2'
Sin.Klk = 0.76828

Tan.Iml = 33÷25 =I.32 Iml = 52°-5I'
Cos.Lml = 0.6039
Sec.Lml = I.65589

Sec.lLm = I.25462 lLm = 37°-09'
Cos.lLm = 0.79706
*

Ij=3.5x200 ÷ I55.83 = 4.5

Jk= (3.5x200) - 225 ÷ I83.43 = 2.59

Kl= (3.5x200) - (250+225) ÷ 2II.28=I.064

Lm= .5 ÷0.797 = 0.628

Posts.

Ii = -3.5

Jj = -(3.5x200) + 225 ÷ 225= -2.II

Kk = -(3.5x200) + 250 + 225 ÷ 250 = -0.90

Ll = -(3.5x200) + (275+250+225) ÷ 275 = I.82

Ee = 0

Live Load Stresses:- The live load chord stresses are the same as the dead load chord stresses.

*　　*　　*　　*　　*

Diagonals.

Load advancing from right.

Ij = 28x200 ÷ 8xI55.83 = 4.50 (Load up to j)

Jk = 2Ix200 ÷ 8xI83.43 = 2.86 (Load up to k)

Kl = I5x200 ÷ 8x2II.27 = I.78 (Load up to l)

Lm = IO ÷ 8x0.797 = I.57 (Load up to m)

Mn = 6 ÷ 8x0.797 = o.94 (Load up to n)

Posts.

Jj =-2Ix200 ÷ 225x8=-2.33 (Load up to k)

Kk =-I5x200 ÷ 250x8=-I.5o (Load up to l)

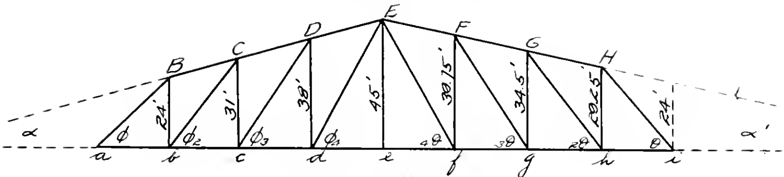
Ll =-IOx200 ÷ 275x8=-0.9I (Load up to m)

Mm by inspection = .75 (Load up to n)

For simplicity we submit the following.

Load up to:-	j	R= 28÷8
	k	R= 2I÷8
	l	R= I5÷8
	m	R= IO÷8
	n	R= 6÷8

ANCHOR SPAN & CANTILEVER ARM.



(Sketch #3)

Considering the above figure, we have a panel length of 25'-0". The pier is located at e. The upper chord rises 7'-0" for all panels between B & E and 5'-3" for all panels between H & E .

The intersection of chords from a =

(24 ÷ 27)25 - 25 = 60.7 Feet.

The intersection of chords from i =

(24 ÷ 5.25)25 = II4.29 Feet.

Tan. α = 24 ÷ 85.7 = .28 α =I5°-39'
Cos. α = .96293

Tan. α_1 = 24 ÷ II4.29 = .2I α_1=II°-52'
Cos. α_1 = .97863

Tan. ϕ = 24 ÷ 25 = .96 ϕ =43°-50'
Cos. ϕ = .72I36
Sin. ϕ = .69256

Tan. θ = 29.25 ÷ 25 = I.I7 θ =49°-29'
Cos. θ = .64967
Sin. θ = .76022

Reaction at i = 4.5

CANTILEVER ARM.

Upper and Lower Chord Stresses.

Hi = 4.5 + 0.76022 = 5.92

hi = -(4.5x25) + 29.25 = -3.85

GH = 4.5x25 + 29.25x.97863 = 3.92

hg = -(4.5x50 - 25 ÷ 34.5 = -7.25

FG = 4.5x50 + 25 + 34.5x.97863= 7.4

fg = -(4.5x75) -25 -50 + 39.75 = -I0.4

EF = 4.5x75+25+50+75 + 39.75x.97863= I0.63

ef = -(4.5xI00) -25 -50 -75 + 45 = -I3.33

*　　*　　*　　*　　*

Posts.

Hh = -(4.5xII4.29) + I39.29 = -3.695

Gg = -(4.5xII4.29) - I39.29 + I64.29 =3.995

Ff = -(4.5xII4.29) - I39.29 - I64.29 + I89.29

= 4.32

Ee = Reaction at pier =

(I00+200+200+I00 +I00+200) +50 minus one

panel load = I5

The terms of the above equations are merely substitutes for those in Merriam & Jacoby's formula.

Diagonals.

Tan. θ = 29.25 ÷ 25 = I.I7 θ = 49° -29'
Sin. θ = .76022

Tan. θ_2 = 34.5 ÷ 25 = I.38 θ_2 = 54° - 4'
Sin. θ_2 = .80970

Tan. θ_3 = 39.75 ÷ 25 = I.59 θ_3 = 57° -50'
Sin. θ_3 = .8465)

Tan. θ_4 = 45.00 ÷ 25 = I.8 θ_4 = 60°-57'
Sin. θ_4 = .87420

Lever arm of Hi = II4.29x.76022 = 87.00'

Gh = I39.29x.80970 =II2.75'

Fg = I64.29x.8465) =I39.00'

Ef = I89.29x.87420 =I65.30'

The stresses then for the diagonals are viz:-

Hi = 4.5xII4.29 ÷-87 = 5.92

Gh = 4.5xII4.29 + I39.29 ÷ II2.75= 5.84

Fg = 4.5xII4.29 + I39.29 + I64.29 ÷ I39

= 5.88

Ef = 5.88 + (I89.29 ÷ I65.3) = 6.09

Thus far, all stresses have been obtained by the method of moments. In order to obtain accuracy,however, many of the stresses have been checked by the method of sections and by isolation of joints.

*** ANCHOR SPAN. ***

At the abutment end of the anchor span we have a negative reaction which is equal to R = I00 -(I00 + I00x200 ÷ Ioo) ÷ 50 minus .5 panel load = 4.5

Chord Stresses

ab = -4.5x25 ÷ 24 = -4.69

BC = 4.5x25 ÷ 24x.963 = 4.87

bc = -4.5x50 -25 ÷ 3I = -8.06

CD = 8.06 ÷ .963 = 8.38

cd = -4.5x75 - 25 - 50 ÷ 38 = -I0.86

DE = I0.86 ÷ .963 = II.29

de = -4.5 -25 - 50-75 ÷ 45 = -I3.32

Posts in Anchor Span.

Bb = -4.5x60.7 ÷ 85.7 = -3.I85

Cc = -4.5x60.7 -25 ÷ II0.7 = -2.7

Dd = -4.5x6).7 -25 -50 ÷ I35.7 = -2.57

Diagonals.

$Tan.\phi_2 = 31 \div 25 = 1.24$ $\phi_2 = 51^\circ - 7'$
$Sin.\phi_2 = .77843$

$Tan.\phi_3 = 38 \div 25 = 1.52$ $\phi_3 = 56^\circ-40'$
$Sin.\phi_3 = .83549$

$Tan.\phi_4 = 45 \div 25 = 1.80$ $\phi_4 = 60^\circ-57'$
$Sin.\phi_4 = .87420$

Lever arm of Cb = 85.7x.77843 = 66.8'

Dc =110.7x.83549 = 92.5'

Ed =135.7x.87420 =118.5'

Stresses.

$aB = 4.5 \div .69256 = 6.5$ Sin. = .69252

$Cb = 4.5x60 + 25 \div 66.8 = 4.47$

$Dc = 4.5x60.7 + 25 + 50 \div 92.5 = 3.77$

$Ed = 4.5x60.7 + 25 + 50 + 75 \div 118.5 = 3.57$

* * * * * *

LIVE LOAD STRESSES IN ANCHOR SPAN.

Chord Stresses.

Reaction when suspended and cantilever arms are fully loaded.

$R= -(\overline{100}^2 + 200x100 \div 100) - (100 + 200) \div 50$

$= -6$

$Cos.\alpha = .96293$

The method of procedure used in the analysis of stresses for a simple truss may be applied here.

ab = -6x25 ÷ 24 = -6.25

BC = 6x25 ÷ 24x.963 = 6.50

bc = -6x50 ÷ 3I = -9.68

CD = 9.68 ÷ .963 = I0.05

cd = -6x75 ÷ 38 = -II.84

DE = II.84 ÷ .963 = I2.2)

de = -6xI00 ÷ 45 = *I3.32

The effect of loading only the anchor span is the reversal of stresses in the upper and lower chord of same. The live load chord stresse under this condition are:-

ab = I.5x25 ÷ 24 = I.56I

BC = -I.56I ÷ .963 = -I.623

bc = I.5x50 - 25 ÷ 3I = I.6I2

CD = -I.6I2 ÷ .963 = -I.68

cd = I.5x75 -25 -50 ÷ 38 = .988

DE = -.988 ÷ .963 = -I0.26

de = I.5xI00 - 25 - 50 - 75 ÷ 45 = 0

Web Stresses in Anchor Span.

Load at:-	b	R = $\frac{3}{4}$
	c	R = I/2
	d	R = I/4
Rest of Bridge		R = -6

aB (Min) = -I.5 ÷ .693 = -2.I68

aB (Max) = 6 ÷ .693 = 8.67

Bb (Min) = I.5x60.7 ÷ 85.7 = I.062

Bb (Max) = -6x60.7 ÷ 85.7 = -4.284

Cb (Min) = -.75x6).7 ÷ 66.8 = -.683

Cb (Max) = 5.25x60.7 + 85.7 ÷ 66.8 = 6.07

Cc (Min) = .75x60.7 ÷ II0.7 = .4I2

Cc (Max) = -5.25x60.7 - 85.7 ÷ II0.7 =-3.66

Dd (Min) = .25x60.7 ÷ I35.7 = .II2

Dd (Max) = -4.75x60.7 + 85.7 + II0.7 ÷ I35.7

= -3.58

Ed. (Min) = 0

Ed (Max) = 4.5x60.7+85.7+II0.35+I35.7 ÷ II8.5

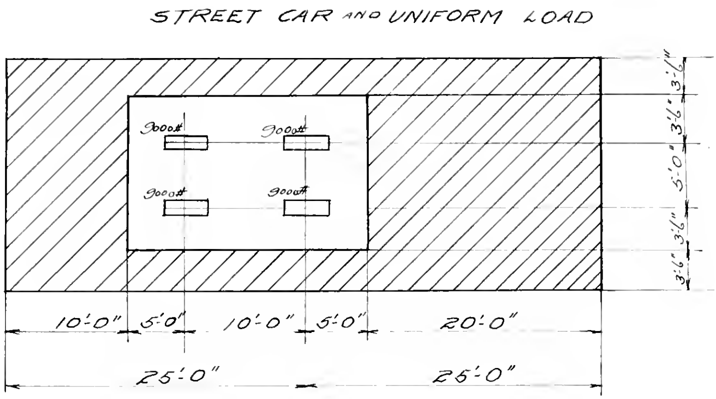
= 5.II

Ee (Min) =-I8 + I = -I7

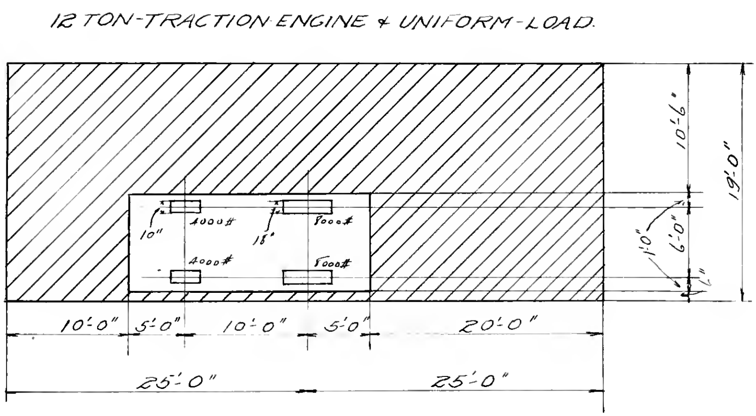
Ee (Max) =-I6 + I = -I5

Cd (Min) = .25x60.7 ÷ 92.5 =-.I65

Cd (Max) = 4.75x60.7+85.7+II0.7 ÷ 92.5 = 5.25



(Sketch #4)



(Sketch #4a)

Planking:-

The total thickness of planking is to be 5.5 inches, I.5 inches of this total to be wearing surface. This wearing surface is to be of oak and capable of being replaced. The thickness to resist bending then,will be 4 inches. The allowable stress for oak = I200/sq.in.. The weight of planking is 4.5#per board foot. The specifications(Art.I8) state that stringers shall be spaced not farther in feet than the thickness in inches of the floor.

Let L = distance between two roadway stringers.

Let b = breadth of planking in in..

" d = depth of planking in inches.

Then,for maximum mending moment place one rear wheel of traction engine half way between two stringers.

The calculations from here on will be self-explanatory. We will endeavor to design the floor sytem so that it will represent a complete analysis.

Resisting Moment = SI/c= I200bd ÷ 6 = 38,400"#

Bending Moment = 4000xL/2 - 4000x4.5 =

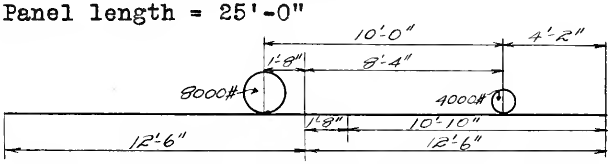
2000L - I800 = 38,400

L = 28.2

In view of this analysis we will space stringers 2'-3" apart.

DESIGN OF ROADWAY STRINGERS.

Panel length = 25'-0"



B. M. =(8000xI4.I7+4000x4.I7 ÷25)I0.83xI2 =

678,000"#

For stringers use I5" I-Beam =	42#
Planking = 5.5x4.5x2.25 =	55.7
Total per Lin. Ft.	97.7#

Dead load bending moment=

$97.7\text{x}\overline{25}^2\text{x}12 \div 8 = 91,500''\#$

Bending Moment caused by wheel loading=	678,000
" " " " dead load	= 9I,500
Total	769,500"#

I/c = 769,500 ÷ I3000 = 59.2

Section Modulus of a I5"-42# I-Beam=58.9

A I5"-42# I-Beam is O.K. but it is perhaps better design to use a I5"-45# I-Beam. S.C.=60.8

*** TRACK - STRINGERS ***

B.M. =(I5x9000+5x9000÷25)I0xI2=8I5,000"#

Planking=5.5x4.5x2.5=	62#	per	lin.	ft.
Track Rails =	30#	"	"	"
I8" I-Beam Stringer =	55#	"	"	"
Total	I47#	"	"	"

D.L.B.M.=wl/8=I47x25xI2 ÷ I2 = I39,000"#

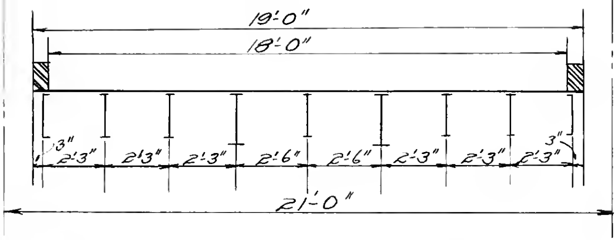
Bending Moment from the car wheel =	8I5,000"#
" " " dead load =	I39,000"#
Total =	954,000"#

$$\frac{954,000}{13,000} = 73.5$$

The I8" 55# I-Beam assumed is O.K.

Its section modulus = 8I.5

DESIGN OF FLOOR BEAM.



Weight of Planking=5.5x4.5x25xI9=II,750

Weight of Steel:-

2-I5" Channels	@ 33#	=	66#
5-I5" I-Beams	@ 45#	=	225#
2-I8" I-Beams	@ 55#	=	II0#
2-90# Rails	@ 30#	=	60#
Total =			46I#/lin.ft.

Weight of the steel per panel:-

46Ix25 = II,525

Planking= II,750#

Steel = II,525#

Fl.Beam= 2,4I5# (II5x2I)*

Total = 25,690#

D.L.B.M. = 25,690x2IxI2/8 = 8I0,000"#

II5# is the assumed weight of Floor Beam per

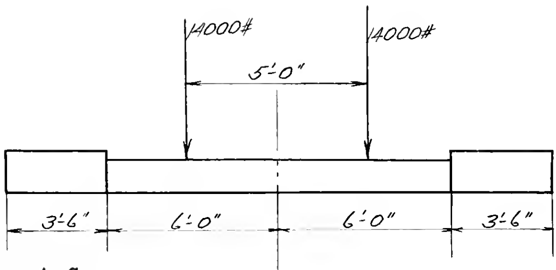
Lin. Ft.

Live Load Stresses for Floor Beam

There is a chance of for two kinds of load-ing.

Ist.) Street car and uniform load.

2nd.) Traction engine and uniform load.



Street Car:-

Each wheel load = 9000#

Wheel load reaction=9000+9000xI5 ÷ 25=I4,400#

The uniform load reaction per foot of floor beam from loading ahead and behind street car = I0xI00x5+20xI00xI5 = I000#

On sides of car per Ft. of Fl. Beam = 25xI00 = 2500#

Fl.Beam reaction =	3.5x2500	=	8750
	6xI000	=	6000
Total=			I4750#

Street car load = I4,400#

Total reaction =I4,400 + I4,750 = 29,I50#

Taking moments at center:-

M=29150x11.5-8750x7.75-6000x3-14400x2.5=184262'#

184,262'# x 12 = 2,211,144"#

Live load Moment= 2,211,144

Dead load Moment= 810,000

Total moment= 3,021,144"#

From this it may be noted that a built up section is required.

Use 2Ls 5 x 3.5 x 11/16	Area =	10.74	sq.	in.
For rivet holes deduct		1.38	"	"
	Net Area =	9.46	"	"

Web of floor beam taken as 24"

Effective depth = 24 - (2x1.72) =20.56"

Area of metal required:-

$$\frac{3,021,144}{13,000} = 9.2$$

The assumed section is hence O. K.

Weight of section per lin. ft. =

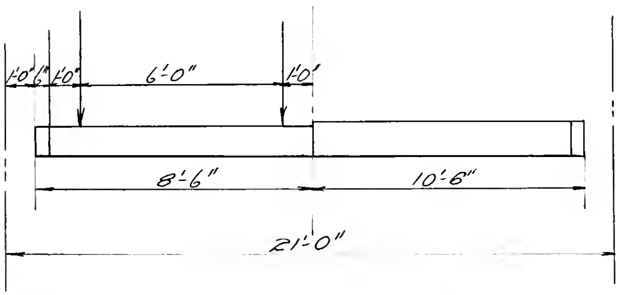
4Ls 5 x 3.5" x 11/16 @ 18.3#	= 73.2
Web Plate 24"x1/2"	= 40.8
Total	114.0#

Assumed section weight = 115# Hence O. K.

For investigating shear, the traction engine will be set against the wheel guard with the rear wheel on floor beam.

Traction engine assumed to cover a space

8' x 20'



Wheel load reaction = 8000 + (4000x15 ÷ 25) =

10,400#

Uniform load reaction going to floor beam from ahead and behind engine:-

=(10x100x5 + 20x100x15) ÷ 25 = 1000#

Side of traction engine = 25 x 100 = 2500#

R x 21 = 8x1000x15.5 + 2500x10.0x6.5 +

10,400(17.5 + 12.5) = 598,500

Street car loading the gives 29,150 # for reaction of floor beam.

Maximum Stringer Reactions .

Roadway Stringer:-

Reaction=8000+(4000xI5+I0xI00x2.5x5÷25)=I0900

Tr. Stringer:-

Reaction=9000+(9000xI5+I0xIo0x2.5x5÷25)=I4900

Planking=5.5x4.5x2.25=55.7

Roadway Stringer 42.0

Total = 97.7

Again:-

Planking=5.5x4.5x2.5	= 62
Track Stringer	= 55
Rail	= 30
	I47# Total

Dead Load = 98x25/2=I225 (Roadway Stringer)

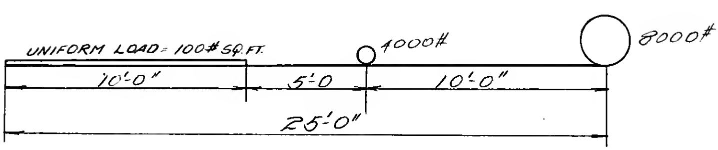
" " = I47x25/2=I838 (Track Stringer)

Total reaction of track Stringer=

I4,900 + I838 = I6,738#

Total reaction of Roadway Stringer=

I0,400 + I,225 = II,625#



LOADS FOR TRUSSES

Live Load:-

See table A of Cooper's Spec. P.IO

Class C Loading.

L. L. IOOO# per ft. of car track.

60# per sq. ft. of remaining surface.

Loading per panel per truss:-

IOOOx25/2	=	I2,500#
Also 60x25/2x(I9-I2)	=	5,250#
Total =		I7,750#

Consider then the live load panel load as

equal to I8,000#

*   *   *   *   *   *   *

A DETERMINATION OF THE DEAD LOAD PANEL WEIGHT PER TRUSS WILL BE MADE: these weights will be determined from a series of formulas. Certain weights however are known and they are

Planking per panel per truss:-

	25 x9.5x5.5x4.5 =	5569#
Wheelguards:-	25x48x4.5 ÷ I2=	450#
Rails:-	=	750#
	Total =	6769#

Steel in Floor:- II.5ft. @ II5#	=	I208
2.5x25x45	=	28I3
I.0x25x33	=	825
I.0x25x55	=	I375
Total	=	624I
Connections	=	250
Grand Total	=	6490

Call it 6500#

6800 + 6500 = I3,300 or 532# per. lin.ft.

*   *   *   *   *   *

FORMULA FOR WEIGHT OF BRIDGE BY E. S. SHAW.

$$w = 300 + I + 22b + I/I5bl(I + .00Il)$$

w = weight of steel per lin. ft. of span.

l = length of span i feet.

b = breadth of roadway including sidewalk.

Note:- This formula does not include weight of woodèn floor, but assumes the weight at I0# per sq. ft. of floor surface. The weight of the wooden floor in our case has been figured exactly.

$$w=300+200+22xI9 +I/I5xI9x200(I+.00Ix200)=I,222$$

$$\frac{I222}{2} = 6II\# \text{ per. lin. ft.}$$

6IIx25 = I5275#=Wt.per panel per truss.

I5,275 + 6,800 + 750(Fence)=22,825.

This is the dead load panel weight from Shaw's formula.

*   *   *   *   *   *

WADDEL'S FORMULA.

w = 34 + 22b + 0.I6bl + 0.7I

= 34+4I8 + 608 +I40 = I200#

w= Total dead load in #s per lin. ft. of bridge including flooring, stringers, trusses side-walks and laterals.

l= span in feet.

b = clear width of roadway sidewalk.

I200 ÷ 2 = 600# per lin. ft. of truss.

600x25 = I5000# dead load panel weight.

*   *   *   *   *   *

FROM KETCHUM'S DESIGN OF HIGHWAY BRIDGES.

From the curves,weight of steel in bridge exclusive of fence = 75000# From previous data then our panel load becomes = to I3,000#

This however is for the lightest kind of a highway bridge,and will be neglected here.

From another set of curves in Ketchum's Design of Highway Bridges we get a panel weight totaling 20,000#

*　　*　　*　　*　　*　　*

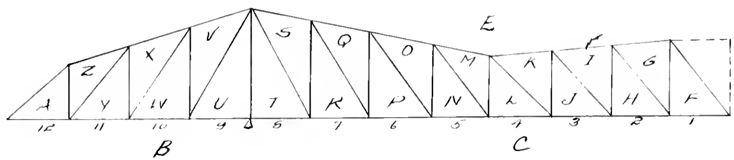
Making the necessary substitutions in formula given by H. G. Tyrrel we get a panel load of I7,000#

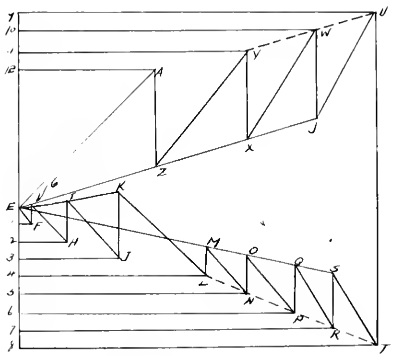
From data obtained from the American Bridge Company, a 200'-0" span through bridge actually up has its two trusses weighing 72,000#. The loading for which the bridge was designed was called Class C.

Using this data panel load becomes

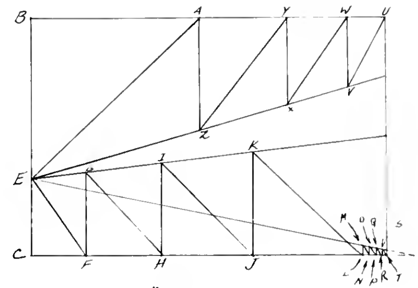
I9,000 #

It appears that from the loading we have assumed that our floor system is very heavy. This therefor increases the dead load. From preliminary trials of sections it has been found that 23,000# per panel would be about the right assumption. This was the result obtained from Shaw's formula.





#I-a



#I-b

PLATES #s I-a & I-b.

EXPLANATION OF PLATES #I-a & #I-b?

Plate #I-a is the graphical solution for the stresses due to dead load of the cantilever bridge during erection. A panel load of unity is considered concentrated at each panel point. There is no need of describing the method of procedure since in drawing the plate no difficulties of any importance arise.

* * * * * *

Plate #I-b is the graphical analysis of the stresses in the members of the cantilever bridge due to the live load of erection. For this, a panel load of unity is considered as concentrated at m. For reasons above stated the method of procedure will not be explained.

* * * * * *

The stresses are tabulated on page ____ The panel load of unity may be considered as one Kip, and therefore the coefficient of the table multiplied by the real panel load will give the stress in the member.

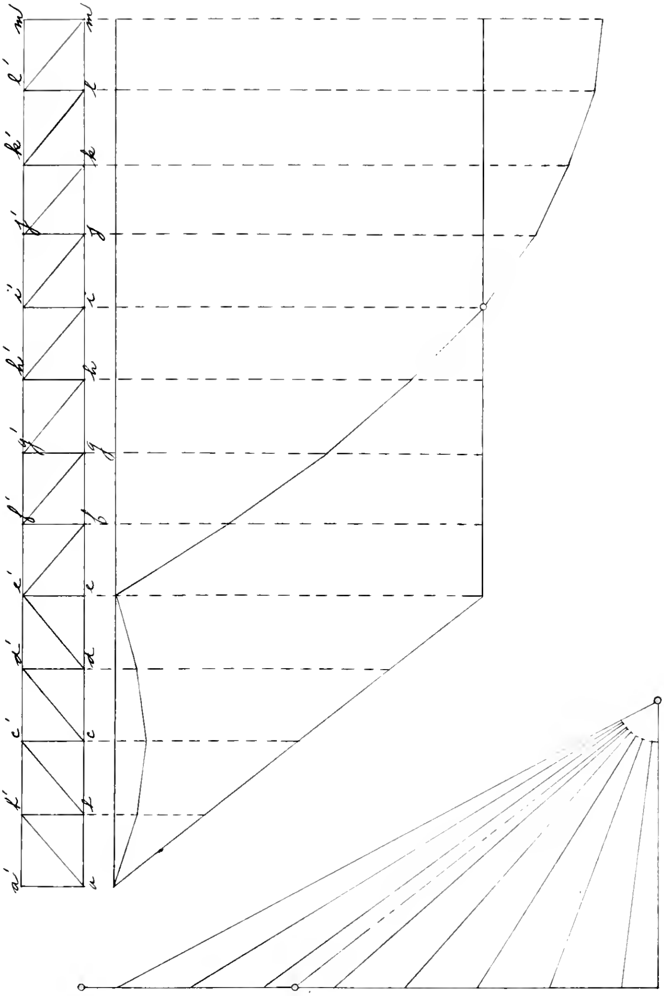


PLATE #2

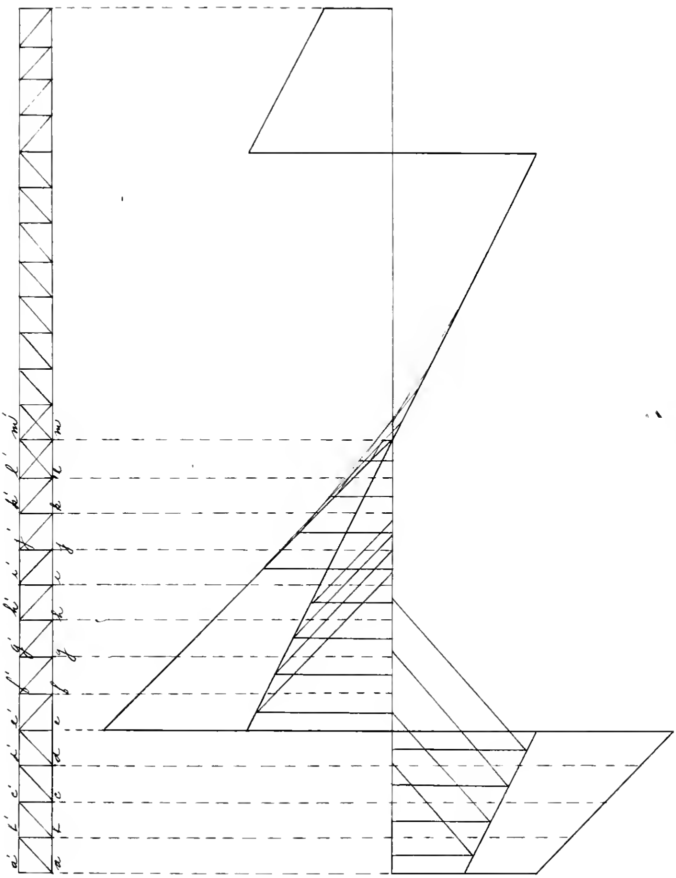


PLATE #3

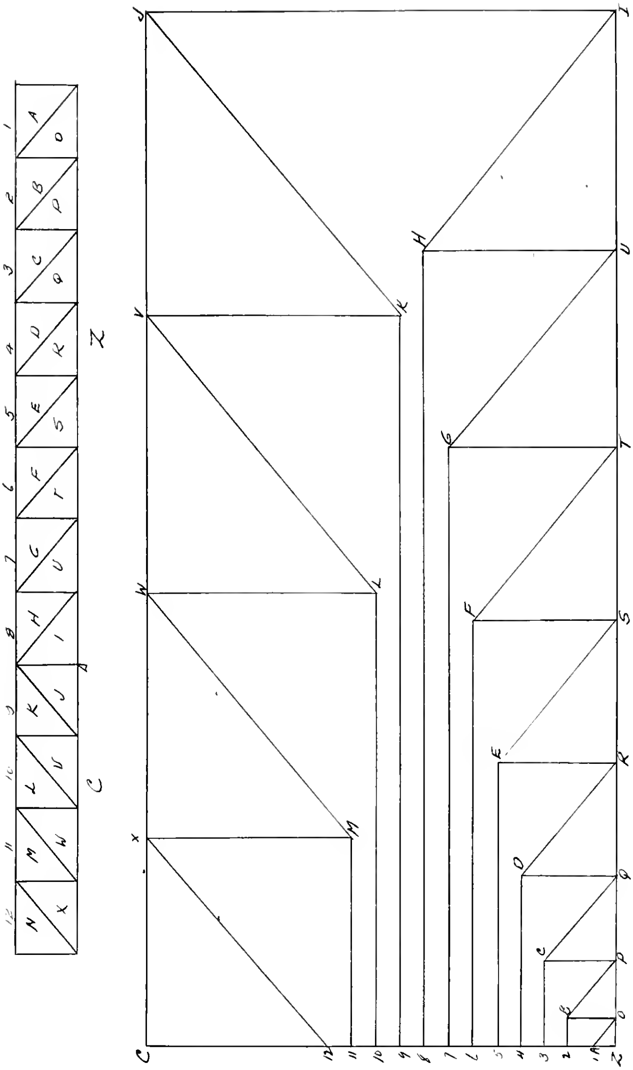


PLATE #4.

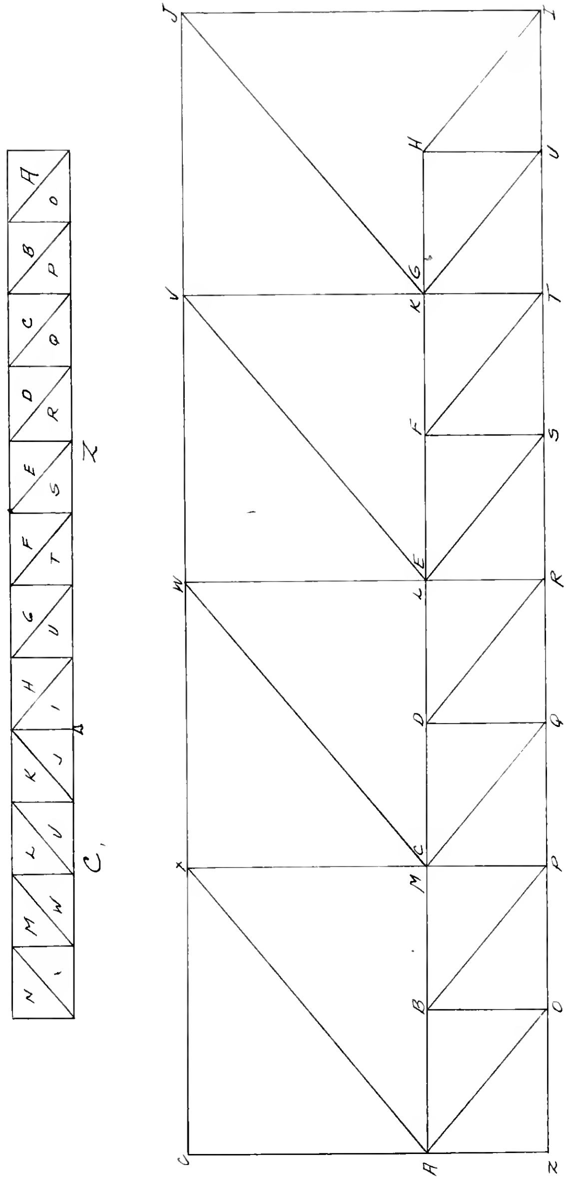


PLATE #5.

EXPLANATION OF PLATES #2 and #3.

It is very necessary that the lateral forces exerted on a bridge be considered. The stresses in the lateral systems of this cantilever bridge will therefore be accurately determined. Art. 39 of Cooper's Specifications says "To provide for wind and vibrations, the top lateral bracing in deck bridges, and the bottom lateral bracing in through bridges, shall be proportioned to resist a lateral force of 300# for each foot of span, I50# Of this to be treated as a moving load. The bottom lateral bracing in deck bridges and the top lateral bracing in through bridges, shall be proportioned to resist a lateral force of I50# per lineal foot. " The determination of stresses in the laterals due to these forces may be solved by graphics in the usual manner.

Plates #2 & #3 are the graphical solution of the stresses in the lateral system after the bridge is in place considering a lateral force of one Kip at each panel point. Strictly speaking

this system is a continuous truss and should be analysed as such. Owing to the complications that arise in solving such a problem it was thought best to consider the system as a cantilever with zero B. M. at I.

Plate"#2" then gives the stresses due to bending moment in the chord members. Plate"#3" gives the stresses in the web members due to shear. In drawing-up theses plates a panel load of one Kip (stated before) was considered. The coefficient then that is obtained multiplied by 3.75 gives the stress in the respective member. By making the assuption stated above the problem reduces to a very simple one.

Plate #4 represents a stress diagram due to the lateral forces during erection.

Plate #5 represents the stress diagram considering a panel load of unity concentrated at the end of the bridge while in erection. Such a loading could perhaps never occur the object having been to compare the drawing with that of Plate #I-b. This truss having parallel chords.

TABLE #I

	BRIDGE UP				BRIDGE IN ERECTION.-				WIND	
	Dead Load		Live Load		Dead Load		Live Load			
	+	-	+	-	+	-	+	-	+Or -	+Or -
aB	6.50		8.67	2.I7	5.40		I.40			
BC	4.87		6.50	I.62	8.09		2.20		I9.05	II.90
CD	8.38		I0.05	I.68	I3.38		3.34		30.38	I9.65
DE	II.92		I2.30	I.03	I7.40		4.08		42.80	28.60
EF	I0.63		I0.63		I8.02		4.50		42.80	28.60
FG	7.40		7.40		I5.70		4.45		33.30	I9.65
GH	3.90		3.90		I3.I5		4.37		25.00	II.90
HI	0		0		I0.70		4.26		I7.85	5.36
IJ		3.26		3.26	5.60		2.80		II.90	0
JK		5.04		5.04	2.44		I.67		7.I4	4.I6
KL		5.23		5.23	.78		.76		3.57	7.I5
LM		6.06		6.06	0		0		I.I9	8.93

TABLE #2.

	BRIDGE UP				BRIDGE IN ERECTION.				WIND.	
	Dead Load		Live Load		Dead Load		Live Load			
	+	-	+	-	+	-	+	-	+or -	+or -
ab		4.69	I.56	6.25		7.80		2.IO	8.9I	5.36
bc		8.06	I.6I	9.68		I2.90		3.20	I9.05	II.90
cd		I8.06	.99	II.84		I6.76		3.94	30.38	I9.65
de & ef		I3.32	0	I3.32		20.00		4.44	42.8	28.60
fg		I0.40		Io.40		I7.60		4.40	33.30	I9.65
gh		7.25		7.25		I5.20		4.35	25.00	II.90
hi		3.85		3.85		I2.82		4.27	I7.85	5.36
ij	0	0	0	0		I0.42		4.I6	II.90	0
jk	3.24		3.24			5.55		2.77	7.I4	4.I6
kl	5.00		5.00			2.50		I.66	3.57	7.I5
lm	5.69		5.69			.75		.75	I.I9	8.93

TABLE #3

	BRIDGE UP.				BRIDGE IN ERECTION.			
	Dead Load		Live Load		Dead Load		Live Load	
	+	-	+	-	+	-	+	-
bC	4.5		6.07	.683	8.20		I.80	
cD	3.77		5.25	.I65	7.20		I.30	
dE	3.57		5.II	0	6.72		I.02	
Ef	6.09		6.09		4.90		.08	
Fg	5.88		5.88		4.50		.I0	
Gh	5.84		5.84		4.I0		.I3	
Hi	5.92		5.92		3.70		.I5	
Ij	4.5		4.5		6.80		I.93	
Jk	2.59		2.80		4.50		I.64	
Kl	I.06		I.78		2.75		I.42	
Lm	.63		I.57		I.25		I.26	
Mn			.94					

TABLE #4.

	BRIDGE UP.				BRIDGE IN ERECTION.			
	Dead Load.		Live Load		Dead Load		Live Load.	
-	+	-	+	-	+	-	+	-
Bb		3.I9	I.06	4.25		5.40		I40
Cc		2.70	.4I	3.68		5.00		I.I0
Dd		2.57	.II	3.58		4.95		0.90
Ee		I5.00		I5.00		I8.50		3.00
Ff		4.32		4.32		3.30		.08
Gg		4.00		4.0		2.80		.09
Hh		3.70		3.70		2.30		.I0
Ii		3.50		3.50		I.80		.I2
Jj		2.II		2.33		3.70		I.33
Kk		.90		I.5		2.32		I.20
lL	I.82			.9I		I.05		I.I
Mm	0			.75		0		.79

TABLE #5.

	BRIDGE UP				BRIDGE IN ERECTION				
	Dead Load		Live Load		Dead Load		Live Load		Maximum.
	+	-	+	-	+	-	+	-	
Ii		70,000		I26,000		25,200		26,000	-I96,000
Jj		42,200		83,900		50,400		26,600	-I26,I00
Kk		I8,000		54,000		32,200		24,000	-72,000
Ll	3,640			32,600		I4,700		22,000	+3,640 -36,700
Mm	0			27,000		0		0	-27,000
Ij	90,000		I62,000		94,400		38,600		252,000
Jk	5I,800		Io3,000		63,000		32,800		I54,800
Kl	2I,280		64,I00		38,500		28,400		85,380
Lm	I2,560		56,500		I8,200		25,200		69,060
IJ		65,200		II7,400	78,500		56,000		+I34,500 -I82,600
JK		I00,800		I82,000	35,700		33,400		+69,I00 -282,800
KL		II4,600		2I2,600	II,200		I5,200		+26,400 -327,200

TABLE #5-Cont'd.

	BRIDGE UP				BRIDGE IN ERECTION				
	Dead Load		Live Load		Dead Load		Live Load		Maximum.
	+	-	+	-	+	-	+	-	
LM		I2I,200		2I8,I00		0		0	-339,300
ij	0		0			I47,000		83,200	-230,200
jk	64,800		II6,700			78,500		55,400	+I8I,500 -I33,900
kl	I00,000		I80,000			35,000		33,200	+280,000 -68,200
lm	II3,000		204,900			I0,500		I5,000	+3I8,700 -25,500
Mn			33,820						33,820
Ml			33,820						33,820

TABLE #6.

	+	+		-	-
ab	8.9I	5.36	a'b'	0	0
bc	I9.05	II.90	b'c'	8.9I	5.36
cd	30.38	I9.65	c'd'	I9.05	II.90
de	42.80	28.60	d'e'	30.38	I9.65
ef	42.80	28.60	e'f'	33.30	I9.65
fg	33.30	I9.65	f'g'	25.00	II.90
gh	25.00	II.90	g'h'	I7.85	5.36
hi	I7.85	5.36	h' i'	II.90	0
ij	II.90	0	i'j'	7.I4	4.I6
jk	7.I4	4.I6	j'k'	3.57	7.I5
kl	3.57	7.I5	k'l'	I.I9	8.93
lm	I.I9	8.93	l'm'	)0	9.52
ab'	II.67	7.00	aa'	7.5o	4.50
bc'	I3.20	8.55	bb'	8.50	5.50
cd'	I4.75	II.I0	cc'	9.50	6.50
de'	I6.35	II.65	dd'	I0.50	7.50
fe'	I2.43	II.65	ee'	I9.50m	I5.00
gf'	I0.90	I0.I0	ff'	8.00	7.50
hg'	9.33	8.55	gg'	7.00	6.50
ih'	7.75	7.00	hh'	6.00	5.50
jh'	6.22	5.44	ii'	5.00	4.50
kj'	4.66	3.79	jj'	4.00	3.50

TABLE #7.

METAL IN SUSPENDED SPAN.				
	Member	Section.	Length	Weight.
2	IJ	2 Channels I5"-35#	25.I8	3525
2	JK	" " " "	25.I8	3525
2	KL	" " " "	25.I8	3525
2	LM	" " I5"-40#	25.00	4000
2	ij	" " I2"-25#	25.00	2500
2	jk	" " " "	25.00	2500
2	kl	" " I2"-30#	25.00	3000
2	lm	" " " "	25.00	3000
2	Ij	" " I2"-25#	34.65	3465
2	Jk	" " I0"-20#	36.80	2944
2	Kl	" " " "	39.I0	3I28
2	Lm	" " " "	4I.50	3320
2	Ii	" " I2"-25#	24.0o	2400
2	Jj	" " I0"-20#	27.00	2360
2	Kk	" " I0"-I5#	30.00	I800
2	Ll	" " " "	33.00	I980
I	Mm	" " " "	33.00	990
			Total:-	47962#

TABLE #8.

METAL IN CANTILEVER & ANCHOR ARMS.				
2	aB	2 Channels I5"-40#	34.80	5568
2	BC	" " I5"-45#	25.95	467I
2	CD	" " I5"-55#	25.95	5709
2	DE	" " " "	25.95	5709
2	EF	" " " "	25.55	5620
2	FG	" " " "	25.25	5620
2	GH	" " I5"-45#	25.55	4599
2	HI	" " I5"-33#	25.55	3373
2	ab	" " I2"-30#	25.00	3000
2	bc	" " " "	25.00	3000
2	cd	" " I2"-35#	25.00	3500
2	de	" " "* "	25.00	3500
2	ef	" " " "	25.00	3500
2	fg	" " " "	25.00	3500
2	gh	" " I2"-30#	25.00	3000
2	hi	" " " "	25.00	3000
			Total.	66869#

TABLE #8 Cont'd.

METAL IN CANTILEVER & ANCHOR ARMS.				
2	bC	2 Channels I2"-30#	39.90	4788
2	cD	2 Channels " "	45.50	5460
2	dE	" " " "	5I.50	6I80
2	Ef	" " " "	5I.50	6I80
2	Fg	" " " "	47.00	5640
2	Gh	" " " "	42.60	5II2
2	Hi	" " " "	38.50	4620
2	Bb	" " I2"-25#	24.00	2400
2	Cc	" " " "	3I.00	3I00
2	Dd	" " " "	38.00	3800
2	Ff	" " " "	39.75	3975
2	Gg	" " " "	34.50	3450
2	Hh	" " " "	29.25	2975
		Total		57680#

FIGURING PANEL WEIGHTS.

From Table #7 the weight of sections is 47,962# This of course excludes the details. For this a percentage of 40% is to be added. The panel weight of the suspended span =(47,962 ÷ 8 + 40%) + 14000 = 22, 393# This agrees very closely with the panel eight assumed.

For the cantilever and anchor spans the total weight of steel in sections = ((135698 + 40%) ÷ 16) + 14000 = 26,000# This was the panel weight used in designing members of cantilever and anchor arms.

14000 is a constant throughout each panel of the bridge, it being the weight of the floor system.

The 40% which was added for details was obtained from sets of figures for bridges actually in place. In tentatively designing a member complete it was found that the details very closely checked this figure.

- USE OF TABLES.-

The stresses due to live & dead load when bridge is in place are obtained by multiplying the coefficient of a member (Tables #1,2,3,&4) by its respective D. L. or L. L. panel weight. Thus to obtain the live load stress in the mem- Kl (taken at random) we find its coefficient in table #3 and multiply it by the Live Load Panel weight,viz:-

I.78 x I8000 = 32,040#

This reduced to an equivalent D. L. bases = 2 x 32,040 = 62,I00# This stress is recorded in table #5.

Similarly the D. L. & L. L. erection stresses are respectively

2.75 x I4000 = 38,500# and

I.42 x 20,000 = 28,400#

The stresses which occur when bridge is up do not occur while bridge is in erection. Then

2I,280 + 64,I00 = 85,380# or Max. Stress.

and this is recorded in column #II of Table #5

For this stress the member is designed.

Designing of Members.

Designing members offers no special difficulties. For that reason the calculations pertaining to the design of sections have been omitted. We do however submit the design of the Post Ee showing how we obtained the section. In a similar manner all members were designed.

Maximum stress in the post Ee occurs when there is a maximum reaction at the pier. This maximum reaction takes place when there is a full live load on the bridge.

Cofficient for D. L. = I5

Coefficient for L. L. = I5

Dead Load panel load in suspended span = 23,000#

Dead Load panel load in cantilever and anchor spans = 36,000#

Live load panel load throughout bridge= I8,000#

Stress in Ee =(I5 x 72,000) - 7 x I6,000 =

968,000#

An approximate r= 7.3

Area required= 968,000÷ I3,400 = 72.25sq.in.

(P= 20,000 -(90x45xI2 ÷ 7.3) = I3,400#)

<u>Section to be tried.</u>

2 Pls. I9"x 3/8"	=	I4.24sq. in.
4 Ls 4"x4"xII/I6"	=	20.I2
2 Pls. 20"xII/I6"	=	27.52
2 Pls. II"xI/2"	=	II.00
Total=		72.88sq. in.

To find r then

I/I2xI9x.375 + 7.I2 x 9.875	=	695
2 x 7.I7 + I0.06 x(8.375)	=	7I5
I/I2 x II/I6 x (20)	=	458
I/I2 x I/2 x (II)	=	56
Total.		I924

I924 x 2 = 3848

r= 7.28

From this it is noted that assumed section is O. K.

CONCLUDING REMARKS.

The stresses due to wind have been omitted in designing the members for under specifications we are allowed to do so. They exceed nowhere 30% the maximum stains due to the dead and live loads upon a member. Regarding the design of portals it may be stated that they were considered as part of the "details" for which a percentage was allowed. This being 40%.

As the bridge now stands the hinge is at i in the unloaded chord. The same could have been placed at I of the "loaded" chord putting in the member iJ. By using this member the post Ii becomes zero and Ij drops out. Furthermore IJ and HI become = to zero. The member ij takes then a stress = to $3.5 \times 25 \div 27 = 3.24$, (Coefficient) for both dead and live load. $3.5 \div .73373 = 4.775$ (Coefficient) stress in iJ (Post) . One Kip equal the stress in Jj.

All other stresses throughout the bridge remain unchanged.

* * * * * *

The reason for putting the hinge in the upper chord may be explained by stating that the appearance of the bridge was considered. By so doing a more pleasing outline for the structure being obtained.

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3) Transport across Sydney Harbor.

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